Pymble L.C. 2004 Ext 2
Trial HSC

Question 1. Use a SEPARATE Writing Booklet — Marks

(a) Find $\int 3x\sec^2(x^2)dx$ [2]

(b) Find $\int \frac{e^{-x}}{\sqrt{1-e^{-x}}}dx$ [2]

(c) Find $\int \frac{\tan^6 x}{\sin^5 x}dx$ [2]

(d) Find $\int \cos^{-1}\theta \ d\theta$ [3]

(e) Let $I_n = \int_0^{\pi} x^n \sin x dx$ where n is an integer

(i) Show that $I_n = \pi^n - n(n-1)I_{n-2}$ for $n \geq 2$ [3]

(ii) Hence evaluate I_5 [3]

Question 2 Use a SEPARATE Writing Booklet — Marks

(a) Given that $W = \frac{3+i}{1-2i}$, express the following in the form $a+ib$

(i) $|W|$ [2]

(ii) $\overline{W}$ [1]

(iii) W^{-1} [1]

(b) Illustrate with a diagram and describe in Geometric terms the locus, represented by the following:

(i) $|i-z| = 3$ [2]

(ii) $\frac{\overline{z}z}{2} = \overline{z} + z$ [2]

(c) Let z be a complex number such that $\arg(z)=\theta$, where $\frac{\pi}{2} < \theta < \pi$, and $|z| = 1$

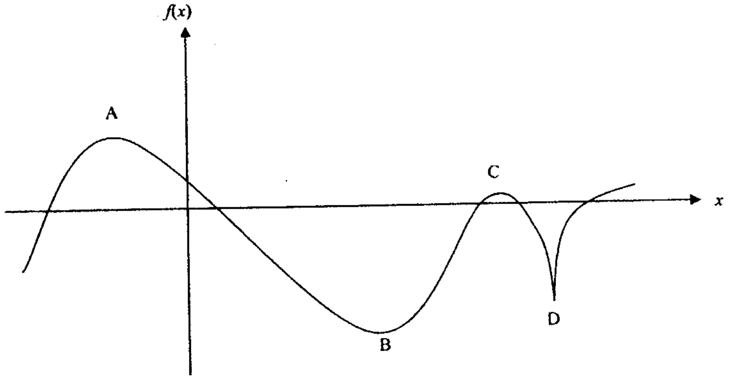
Sketch z^2 and z on an Argand Diagram and find in terms of θ the values of

(i) $\left|\frac{2}{1+z^2}\right|$ [3]

(ii) $\arg\left(\frac{2}{1+z^2}\right)$ [4]

Question 3. Use a SEPARATE Writing Booklet — Marks

(a) On your answer page make a rough copy of the sketch of $f(x)$ below. Label the turning points A, B, C and label the cusp D as indicated. On the same set of axes sketch the derivative function, $f'(x)$ [3]



(b) Sketch the curves on two separate diagrams, show the equations of any asymptotes, and show any intercepts on the axes.

(i) $y = \dfrac{5-x}{x}$ [2]

(ii) $y = \dfrac{25-x^2}{x^2}$ [2]

Hence or otherwise sketch on another two separate diagrams, the curves

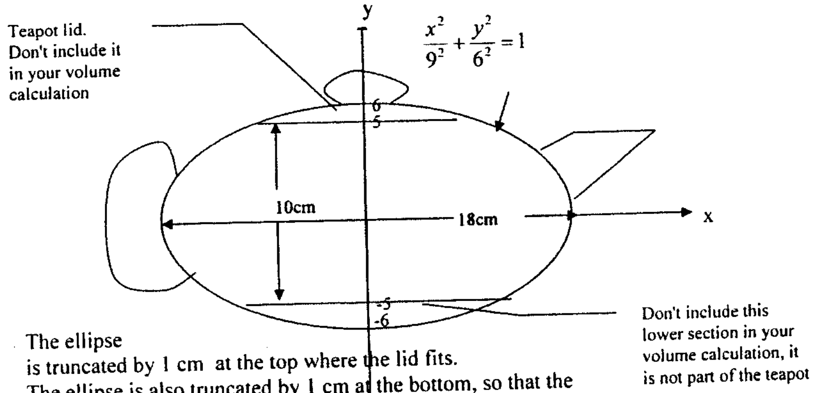
(iii) $y = \left|\dfrac{5-x}{x}\right|$ [2]

(iv) $y^2 = \dfrac{25-x^2}{x^2}$ [3]

(c) Sketch and label $y = \sin^3(2x)$ *for* $0 \le x \le \pi$ and on the same axes sketch and label $y = \ln(\sin^3(2x))$ *for* $0 \le x \le \pi$. [3]

Question 4. Use a SEPARATE Writing Booklet — Marks

(a) A large shiny metal teapot appears to be circular when viewed from above. The same teapot appears to be elliptical when viewed from the side (as shown). [6]



The ellipse is truncated by 1 cm at the top where the lid fits. The ellipse is also truncated by 1 cm at the bottom, so that the teapot has a flat base. This truncated ellipse is rotated about the vertical axis to form the teapot shape.

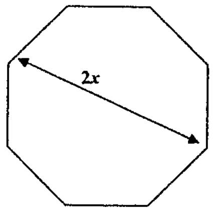
Use the method of shells to find the volume of this shiny metal teapot and then express its capacity in litres correct to 2 decimal places.

(b) With the aid of a sketch and careful integration show that the area enclosed by the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is πab square units. [3]

Question 4 continues next page

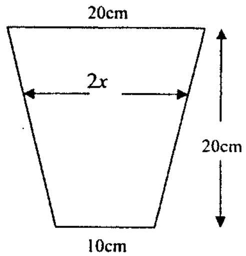
Question 4. continued

(c) The horizontal cross-section of a vase is a regular octagon. The maximum width $2x$ of the octagonal cross-section is 10cm at the base and 20cm at the top.



The horizontal cross-section is shown opposite

The vertical cross-section is shown opposite.



(i) Find an expression for the area of the horizontal cross-section when the maximum width is $2x$ [2]

(ii) Find the volume of the vase using the method of **parallel cross-sections**. [leave your answer in cm^3] [4]

Question 5. Use a SEPARATE Writing Booklet Marks

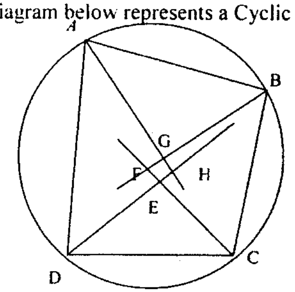
(a) A square-based pyramid, of base length, s and perpendicular height, h is inscribed in a sphere of radius r.

(i) Show that $s^2 = 4hr - 2h^2$ [2]

(ii) Find h in terms of r so that the pyramid has maximum volume. [3]

(iii) Find an expression for the maximum volume in terms of r. [2]

(b) The diagram below represents a Cyclic Quadrilateral $ABCD$.



Bisectors of the angles have been drawn, forming a smaller quadrilateral $EFGH$. Copy the diagram onto your answer page and Prove that $EFGH$ is also a Cyclic Quadrilateral [5]

(c) Solve for x

$\tan^{-1}5x - \tan^{-1}3x = \tan^{-1}\frac{1}{4}$ [3]

Question 6. Use a SEPARATE Writing Booklet — Marks

(a). (i) Express the complex cube roots of unity, ω *and* ω^2 in the form $rcis\theta$ [1]

(ii) Using Argand diagram or otherwise, show that $\omega + \omega^2 + \omega^3 = 0$ [3]

(iii) Prove that if $P(x) = p_0 + p_1x + p_2x^2 + ... + p_nx^n$ has a root of multiplicity m, then $P'(x)$ has a root of multiplicity (m -1) . [3]

(iv) Prove that ω, a complex cube root of unity is a repeated root of the polynomial $P(x) = 5x^5 + 7x^4 + 9x^3 + x^2 - x - 3$ [4]

(b) If α, β, γ are the roots of $x^3 - 9x + 9 = 0$,

show that $(\alpha - 1)(\beta - 1)(\gamma - 1) = -1$ [4]

Question 7. Use a SEPARATE Writing Booklet — Marks

(a) (i) Write down the parametric equations that correspond to to the Cartesian equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ [1]

(ii) Show that the greatest area of a rectangle inscribed in an ellipse is $2ab$ u^2 [2]

(b) For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ find

(i) Eccentricity [1]

(ii) Coordinates of the foci [1]

(iii) Equations of the directrices [1]

(iv) Equation of the tangent to the hyperbola at the point $(5, -2\frac{1}{4})$ [2]

(v) Show foci, directrices, asymptotes and the tangent on a neat sketch. [4]

(c) An hyperbola has asymptotes $y = \pm x$ and it passes through the point (-3,-2).

(i) Find the equation of the hyperbola [1]

(ii) Find the length of the transverse axis [1]

(iii) Explain why this hyperbola is rectangular [1]

Question 8. Use a SEPARATE Writing Booklet — Marks

(a) The positive integers are bracketed as follows:
(1), (2,3), (4,5,6), ...
where there are r integers in the r^{th} bracket.
Prove that the sum of the integers in the r^{th} bracket is $\frac{1}{2}r(r^2+1)$. [5]

(b) If $a>0$, $b>0$, $c>0$, show that $a^2+b^2+c^2 \geq ab+bc+ca$
and state the condition of equality. [3]

(c) (i) Express $1+x+x^2+x^3+x^4+x^5$ as a product of real factors [1]

(ii) Prove that the equation $c+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}+\frac{x^6}{6}=0$
has no real roots if $c>\frac{37}{60}$ [6]

END OF PAPER

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0, \text{ if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\quad -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x,\quad x>0$

Q1 a) $= \frac{3}{2}\int 2x \sec^2(x^2)\,dx$

$= \frac{3}{2}\int \sec^2(x^2)\,dx^2$ } 1 } 2 E_8 2

$= \frac{3}{2}\tan(x^2) + c$ 1

b) $= \int (1-e^{-x})^{-\frac{1}{2}} d(1-e^{-x})$ $\frac{d}{dx}(1-e^{-x}) = e^{-x}$ 1 } 2 E_8 2

$= 2(1-e^{-x})^{\frac{1}{2}} + c$ 1

c) $= \int \frac{\sin x}{\cos^6 x}\,dx$

$= -\int \cos^{-6}x\, d\cos x$ } 1 } 2 E_8 2/3

$= -\frac{\cos^{-5}x}{-5}$

$= \frac{1}{5\cos^5 x} + c$ } 1

d). $\int \cos^{-1}\theta\, d\theta =$

$= \theta.\cos^{-1}\theta - \int \theta . \frac{-1}{\sqrt{1-\theta^2}}\,d\theta$ 1

$= \theta.\cos^{-1}\theta - \frac{1}{2}\int (1-\theta^2)^{-1/2}\, d(1-\theta^2)$ 1 } 3 E_8 3.

$= \theta.\cos^{-1}\theta - \frac{1}{2}.\frac{(1-\theta^2)^{\frac{1}{2}}}{\frac{1}{2}}$ } 1

$= \theta.\cos^{-1}\theta - \sqrt{1-\theta^2} + c$

e)(i) $I_n = \int_0^\pi x^n \sin x\,dx$.

$\therefore I_n = \int_0^\pi x^n\, d\cos x$ } 1

$= [-x^n\cos x]_0^\pi - \int_0^\pi \cos x . -nx^{n-1}\,dx$ } 3 E_8 4

$= (-x^\pi . -1) + \int n\,x^{n-1}\, d\sin x.$ } 1

$= x^\pi + [n x^{n-1} \sin x]_0^\pi - \int_0^\pi \sin x . n.(n-1)\,x^{n-2}\,dx$ } 1

$= x^\pi - n(n-1)\,I_{n-2}$ for $n \geq 2$

Q1 ctd.

e(ii) $I_5 = \pi^5 - 20 I_3$

$= \pi^5 - 20(\pi^3 - 6I)$ } 1

$= \pi^5 - 20\pi^3 + 120\int_0^\pi x\sin x\,dx$ } 1 } 3 E_8 2

but $\int x\sin x\,dx = \int x\, d(-\cos x)$

$= [-x\cos x]_0^\pi + \int_0^\pi \cos x\,dx$

$= \pi + [\sin x]_0^\pi$ } 1

$= \pi$

$\therefore I_5 = \pi^5 - 20\pi^3 + 120\pi$.

Q2 a)(i) $W = \frac{3+i}{1-2i} \times \frac{1+2i}{1+2i}$

$= \frac{1+7i}{5}$ } 1 } 2 E_3 2

$\therefore |W| = \sqrt{\frac{1}{25} + \frac{49}{25}}$

$= \sqrt{2}$ } 1

(ii) $\bar{W} = \frac{1}{5} - \frac{7i}{5}$ 1   "   "

(iii) $\frac{1}{W} = \frac{\bar{W}}{W\bar{W}}$

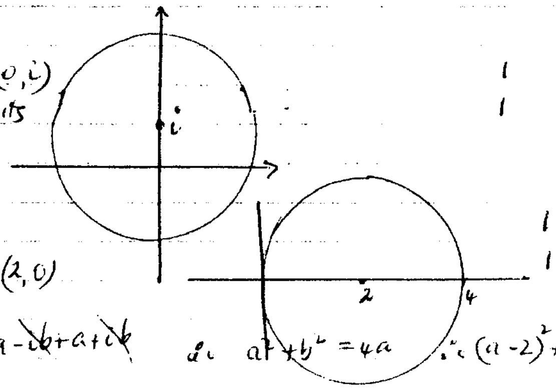
$= \frac{\bar{W}}{|W|^2}$

$= \frac{1-7i}{5} \times \frac{1}{(\sqrt{2})^2}$ } 1   "   "

$= \frac{1}{10} - \frac{7i}{10}$

b)(i) circle centre $(0,i)$ 1 E_3 3

radius 3 units   1



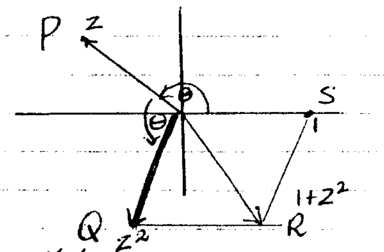
(ii) circle centre $(2,0)$ 1 E_3 3

radius 2.   1

working: $\frac{a^2+b^2}{2} = a - ib + a + ib$ i.e. $a^2 + b^2 = 4a$ $\therefore (a-2)^2 + b^2 = 4$.

Q 2 ctd.

c)



algebraic method

(i) $1+z^2 = 1+\cos 2\theta + i\sin 2\theta$

$\therefore |1+z^2| = \sqrt{1+2\cos 2\theta+\cos^2 2\theta + \sin^2 2\theta}$

$= \sqrt{2+2\cos 2\theta}$ } 1

$= \sqrt{2(1+2\cos^2\theta - 1)}$

$= \sqrt{4\cos^2\theta}$ } 1

$= 2\cos\theta$

$\therefore \left|\dfrac{2}{1+z^2}\right| = \dfrac{|2|}{|2\cos\theta|}$ } 1

$= \sec\theta$

3 E_3 3

geometric method

OQRS is a rhombus, side length 1 unit.

$Q\hat{O}S = 360° - 2\theta$ (angles at a point)

$O\hat{Q}R = 180° - (360° - 2\theta)$ (cointerior angles, OS ∥ QR)

$= 2\theta - 180°$

In $\triangle OQR$

$OR = |1+z^2|$

$= q$

and $q^2 = r^2 + o^2 - 2\times r\times o\times\cos(2\theta - 180°)$

$= 1 + 1 - 2(\cos 2\theta\cos 180° + \sin 2\theta\sin 180°)$

$= 2 - 2(-\cos 2\theta)$

$= 2 + 2\cos 2\theta$

$= 4\cos^2\theta$

$\therefore q = 2\cos\theta$

$\therefore \left|\dfrac{2}{1+z^2}\right| = \dfrac{2}{2\cos\theta}$

$= \sec\theta$

Q2 ctd. geometric method.

c(ii) OR bisects $Q\hat{O}S$

$= \frac{1}{2}(2\pi - 2\theta) = \pi - \theta$ } 1

$\therefore \arg(1+z^2) = -(\pi - \theta)$ [principal argument] } 1

$= \theta - \pi$

Now $\arg\left(\dfrac{2}{1+z^2}\right) = \arg 2 - \arg(1+z^2)$ } 1

$= 0 - (\theta - \pi)$ } 1

$= \pi - \theta$

4 E_3 3

or algebraic method

$\arg\left(\dfrac{2}{1+z^2}\right) = \arg 2 - \arg(1+z^2)$

$= 0 - \arg(1+\cos 2\theta + i\sin 2\theta)$ } 1

$= -\tan^{-1}\left(\dfrac{\sin 2\theta}{1+\cos 2\theta}\right)$

$= -\tan^{-1}\left(\dfrac{2\sin\theta\cos\theta}{\cancel{1}+2\cos^2\theta\cancel{-1}}\right)$ } 1

$= -\tan^{-1}(\tan\theta)$

but θ is in 2nd quadrant and } 1

$\tan^{-1}x$ is defined: $-\frac{\pi}{2} < \tan^{-1}x < \frac{\pi}{2}$

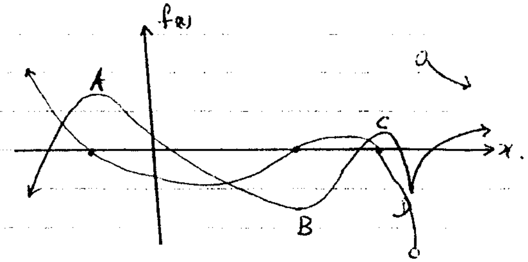
$\therefore \arg\left(\dfrac{2}{1+z^2}\right) = -\tan^{-1}(-\tan(\pi-\theta))$

$= -\tan^{-1}\tan(\theta - \pi)$ } 1

$= -(\theta - \pi)$

$= \pi - \theta$

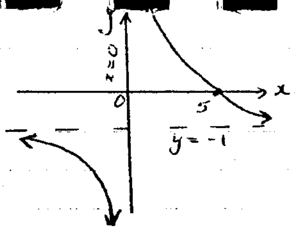
Q3 a)



} 3 E_6 3

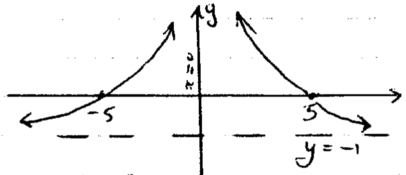
Q3 ctd

b) (i) $y = \frac{5}{x} - 1$



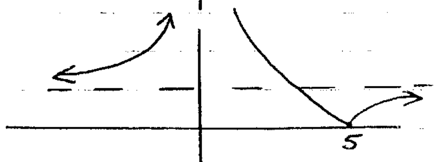
}2 E6 3

(ii) $y = \frac{25}{x^2} - 1$



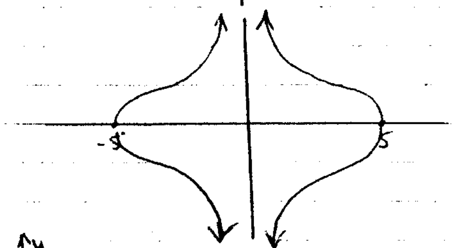
}2 " "

(iii)



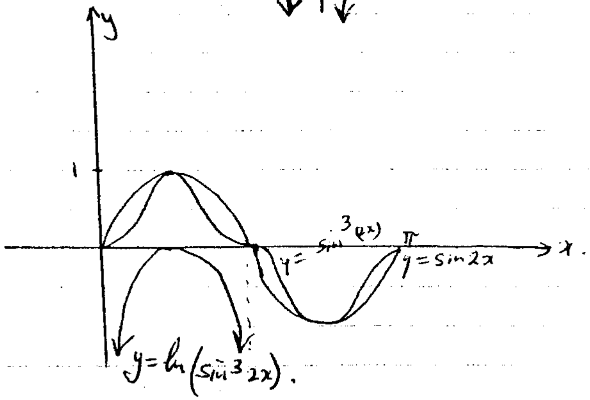
}2 " "

(iv)

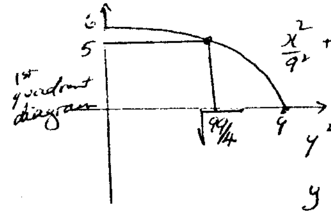


}3

c)



Q4 a)



$\frac{x^2}{9^2} + \frac{y^2}{6^2} = 1$

$y^2 = \frac{6^2}{9^2}(9^2 - x^2)$

$y = \pm \frac{6}{9}\sqrt{9^2 - x^2}$ } 1

Typical Shell

$\delta V = 2\pi x y \,\delta x$ } 1

Total Volume

$= 2 \times 2\pi \left[\int_0^{\sqrt{99/4}} 5x\,dx + \frac{6}{9}\int_{\sqrt{99/4}}^{9} x \cdot \sqrt{9^2 - x^2}\,dx \right]$ }1

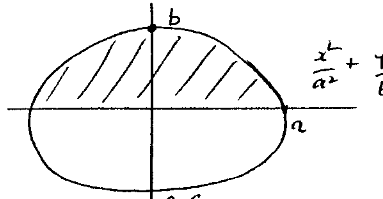
$= 4\pi \left[\left[\frac{5x^2}{2}\right]_0^{\sqrt{99/4}} - \frac{1}{2} \times \frac{2}{3} \left[\frac{(9^2 - x^2)^{3/2}}{3/2} \right]_{\sqrt{99/4}}^{9} \right]$ }1

$= 4\pi \left(\frac{5}{2} \times \frac{99}{4} - \frac{2}{9}\left(\frac{225}{4}\right)^{3/2} \right)$ }1

$= 1955.64 \ldots \text{ cm}^3$ or mL

≈ 1.96 Litres } 1 E7 3

b)



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Total area $= 2\int_{-a}^{a} y\,dx$. }1

Where $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$.

$\therefore$ Area $= 2 \cdot \frac{b}{a} \int_{-a}^{a} \sqrt{a^2 - x^2}\,dx$ }1

but $\int_{-a}^{a} \sqrt{a^2 - x^2}\,dx$ is a semicircular area

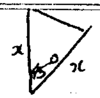
$= \frac{1}{2}\pi a^2$

$\therefore$ area of ellipse $= 2 \times \frac{b}{a} \times \frac{1}{2}\pi a^2$

$= \pi ab \ u^2$. }1 E7 3

Q4 c)

(i) 8 sections



@ Area $= \frac{1}{2} x^2 \sin 45$

$= \frac{1}{2\sqrt{2}} x^2$

$= \frac{\sqrt{2} x^2}{4}$

∴ Total cross-section $= 8 \times \frac{\sqrt{2}}{4} x^2$

$= 2\sqrt{2} x^2 \ u^2$ 2. E_7 2

(ii) $\delta V = A \delta h$

$= 2\sqrt{2} x^2 \delta h$.

Now $2x = \frac{h}{2} + 10$ where $0 \le h \le 20$

ie $x = \frac{h}{4} + \frac{20}{4}$

$\therefore V = 2\sqrt{2} \int_0^{20} \left(\frac{h+20}{4}\right)^2 dh$.

$= \frac{2\sqrt{2}}{16} \int_0^{20} h^2 + 40h + 400 \, dh$.

$= \frac{\sqrt{2}}{8} \left[\frac{h^3}{3} + 20h^2 + 400h\right]_0^{20}$

$= \frac{\sqrt{2}}{8} \left(\frac{8000}{3} + 8000 + 8000\right)$

$= \frac{\sqrt{2}}{8} \times \frac{56000}{3}$

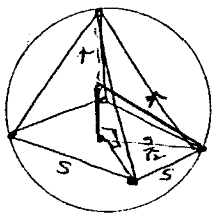
$= \frac{7000\sqrt{2}}{3}$

≈ 3300 cm^3 4. E_7 2/3

Q5 a) (i) $(h-r)^2 + \left(\frac{s}{\sqrt{2}}\right)^2 = r^2$

$\frac{s^2}{2} + h^2 - 2hr + r^2 = r^2$

$s^2 = 4hr - 2h^2$ 2.



(ii) Area of base $= s^2$

Vol of pyramid $= \frac{1}{3} s^2 h$.

$= \frac{1}{3}(4hr - 2h^2)h$.

$\frac{dV}{dh} = \frac{1}{3}(8hr - 6h^2) = \frac{2h}{3}(4r - 3h)$.

$= 0$ for st pts when $h = \frac{4r}{3}$

Q5 a) ctd

(ii) $\frac{d^2V}{dh^2} = \frac{8r - 12h}{3}$ and at $h = 4r/3$

$= -\frac{8r}{3}$

< 0 ∴ max volume when $h = \frac{4r}{3}$ 3 E_6 4.

(iii) ∴ Max Vol $= \frac{1}{3}\left(4 \times \frac{16r^2}{9} \times r - 2 \times \frac{64r^3}{27}\right)$

$= \frac{1}{3}\left(\frac{192 - 128}{27}\right) r^3$

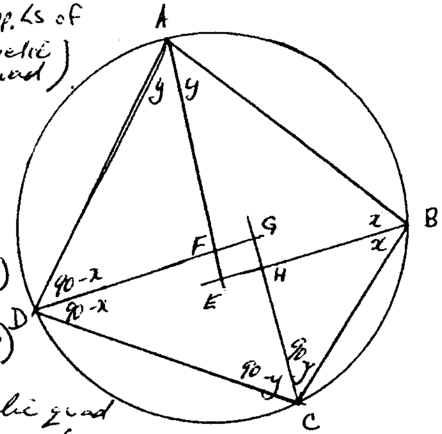
$= \frac{64}{81} r^3 \quad u^3$ 2. E_6 4

b) proofs

Let $A\hat{B}C = 2x$ (opp. ∠s of cyclic quad)

∴ $A\hat{D}C = 180 - 2x$

Similarly $B\hat{A}D = 2y$



In EFGH

$F\hat{G}H = 180 - (90 - x + 90 - y)$

$= x + y$ (∠ sum ∆ DGC)

and $F\hat{E}H = 180 - (x+y)$ (∠ sum ∆ AEB)

∴ EFGH is a cyclic quad

Since $F\hat{G}H + F\hat{E}H = 180$ (opp ∠s supplementary). 5 $E_{1,2}$

c) $\tan(\tan^{-1}5x - \tan^{-1}3x) = \tan(\tan^{-1}\frac{1}{4})$ 5 $E_{1,2}$

LHS $= \frac{5x - 3x}{1 + 15x^2}$ RHS $= \frac{1}{4}$

∴ $8x = 1 + 15x^2$

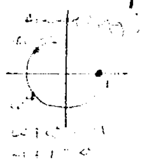
$(5x - 1)(3x - 1) = 0$

$x = \frac{1}{5}, \frac{1}{3}$. 3 E_4

Q6 a)(i) $\omega = \text{cis}\,\frac{2\pi}{3}$

$\omega^2 = \text{cis}\,\frac{4\pi}{3}$ 1 E_3 2

$\omega^3 = \text{cis}\,\frac{6\pi}{3} = \text{cis}\,2\pi = 1$



(ii) $\omega + \omega^2 + \omega^3 = -\frac{1}{2} + \frac{\sqrt{3}i}{2} - \frac{1}{2} - \frac{\sqrt{3}i}{2} + 1$

$= -1 + 1$

$= 0$ 3 E_3 2

(iii) Let $P(x) = (x-\alpha)^m \cdot Q(x)$

where $Q(\alpha) \neq 0$

Then $P'(x) = m(x-\alpha)^{m-1}Q(x) + Q'(x)\cdot(x-\alpha)^m$

$= (x-\alpha)^{m-1}(m\,Q(x) + Q'(x))$

$= (x-\alpha)^{m-1} \cdot R(x)$

$\therefore (x-\alpha)$ is a factor of $P'(x)$ and

α is a root of $P'(x)=0$ of order $m-1$. 3 E_3 3

(iv) If ω is a repeated root

then $P(\omega) = P'(\omega) = 0$

now $P'(x) = 25x^4 + 28x^3 + 27x^2 + 2x - 1$

$\therefore P'(\omega) = 25 \times 1 \times \omega + 28 \times 1 + 27\omega^2 + 2\omega - 1$

$= 27\omega + 27\omega^2 + 27$

$= 27(\omega + \omega^2 + 1)$ from (ii) above

$= 27 \times 0$

$= 0$ 4 E_3 2

b) Since α, β, γ are roots then

$\alpha^3 - 9\alpha + 9 = 0$ ie $\alpha^3 = 9(\alpha - 1)$

$\beta^3 - 9\beta + 9 = 0$ $\beta^3 = 9(\beta - 1)$

$\gamma^3 - 9\gamma + 9 = 0$ $\gamma^3 = 9(\gamma - 1)$

$\therefore \alpha^3\beta^3\gamma^3 = 9^3(\alpha-1)(\beta-1)(\gamma-1)$

but $\alpha\beta\gamma = -9$

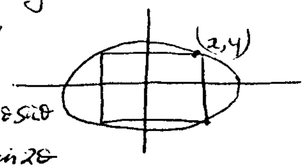
$\therefore$ LHS $= -81$ RHS $= 81(\alpha-1)(\beta-1)(\gamma-1)$

$\therefore (\alpha-1)(\beta-1)(\gamma-1) = -1$ as required 4 E_3 3

Q7.

a) (i) $x = a\cos\theta$ $y = b\sin\theta$ 1 2

(ii) area of rectangle $= 2x \cdot 2y$



$= 4ab\cos\theta\sin\theta$

$= 2ab\sin 2\theta$

$= 2ab$ when $\sin 2\theta = 1$ 2 E_4 2

$2\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

b)(i) $b^2 = a^2(e^2 - 1)$

$e^2 = \frac{b^2}{a^2} + 1$

$= \frac{9}{16} + 1$

$= \frac{25}{16}$

$\therefore e = \frac{5}{4}$ $(e > 0)$ 1 E_4 2

(ii) foci $(\pm ae, 0)$ ie $(\pm 5, 0)$ 1

(iii) $x = \pm \frac{a}{e}$ ie $x = \pm\frac{16}{5}$ 1

(iv) $\frac{2x}{16} - \frac{2y}{9}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{9x}{16y}$

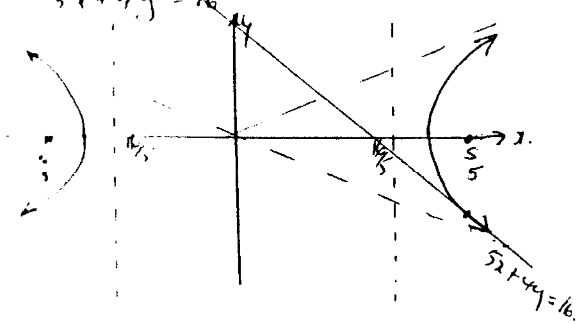
$= \frac{-5}{4}$

eqn of tangent

$y + \frac{9}{4} = -\frac{5}{4}(x - 5)$

$5x + 4y = 16$ 2

(v)

 4

Q7

c) (i) $y = \pm x \Rightarrow a = b = 1$

$\therefore \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

now $(-3,-2)$ satisfies

$\therefore \frac{9}{a^2} - \frac{4}{a^2} = 1$

$\therefore 5 = a^2$

$\therefore$ equation is $\frac{x^2}{5} - \frac{y^2}{5} = 1$ 1 E4 3

note equation can be expressed as $xy = \frac{5}{2}$ (referred to new axes)

(ii) $2\sqrt{5}$ 1 " "

(iii) because asymptotes are perpendicular to each other 1 " "

Q8 By inspection it can be seen that the last digit is a triangular number

$1 \quad 1+2 \quad 1+2+3 \quad 1+2+3+4 \quad \ldots$

summing these, $a=1$, $d=1$ for each bracket $\therefore$ the last digit in the $(r-1)^{th}$ bracket is

$1+2+3+\ldots+(r-1) = \frac{r-1}{2}(1 + r - 1)$

$= \frac{r-1}{2} \cdot r$

now the next consecutive number ie $\left(\frac{r-1}{2} \cdot r + 1\right)$ is the 1^{st} digit in the r^{th} bracket.

But the last digit in the r^{th} bracket

$= \frac{r}{2}(1+r)$

$\therefore$ Sum of integers in r^{th} bracket

$= \frac{r}{2}\left(\frac{r-1}{2} \cdot r + 1 + \frac{r+1}{2} \cdot r\right)$

$= \frac{r}{2}\left(\frac{r}{2}(r - \cancel{1} + r + \cancel{1}) + 1\right)$

$= \frac{r}{2}(r^2 + 1)$ as required 5. $E_{1,2,9}$ 4.

b) $(a-b)^2 = a^2 + b^2 - 2ab$

$a^2 + b^2 \geq 2ab$ equality for $a = b$

Similarly $b^2 + c^2 \geq 2bc$

$c^2 + a^2 \geq 2ca$

$\therefore \cancel{2}(a^2+b^2+c^2) \geq \cancel{2}(ab+bc+ca)$ equality for $a=b=c$ 3 3

8c) (i) $= 1 + x + x^2(1+x) + x^4(1+x)$

$= (1+x)(1+x^2+x^4)$ 1 $E_{1,2,9}$ 4

(ii) Let $f(x) = \frac{x^6}{6} + \frac{x^5}{5} + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c$

then $f'(x) = x^5 + x^4 + x^3 + x^2 + x + 1$

$= 0$ for stationary pts

when $(1+x)(1+x^2+x^4) = 0$

but $1+x^2+x^4$ has no real roots since $x^2 \geq 0$, $x^4 \geq 0$

$\therefore f'(x) = 0$ only for $x = -1$

also

$f''(x) = 5x^4 + 4x^3 + 3x^2 + 2x + 1$

$\therefore f''(-1) = 5 - 4 + 3 - 2 + 1$

$= 3$

$> 0 \quad \therefore$ min^m T.P. when $x = -1$

$\therefore$ for no real roots, need $f(-1) > 0$

ie need

$\frac{(-1)^6}{6} + \frac{(-1)^5}{5} + \frac{(-1)^4}{4} + \frac{(-1)^3}{3} + \frac{(-1)^2}{2} + -1 + c$

ie $\frac{1}{6} - \frac{1}{5} + \frac{1}{4} - \frac{1}{3} + \frac{1}{2} - 1 + c > 0$

$\frac{10 - 12 + 15 - 20 + 30 - 60}{60} + c > 0$

$-\frac{37}{60} + c > 0$

$\therefore c > \frac{37}{60}$ 6 " "